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DEVELOPMENT OF RELIABILITY SAMPLING PLANS FOR TRUNCATED LIFE TESTING UNDER THE EXPONENTIATED GUMBEL DISTRIBUTION

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A B S T R A C T

This paper develops reliability acceptance sampling plans under Binomial, Poisson, and Bivariate Poisson frameworks, assuming lifetimes follow an Exponentiated Gumbel distribution. The proposed plans support effective lot acceptance and rejection decisions, particularly when complete life testing is impractical. Sampling schemes are constructed for various acceptance numbers, confidence levels, and test duration ratios, and the minimum sample size is determined to satisfy specified reliability requirements. Monte Carlo simulation is employed to generate lifetime data and evaluate the performance of the proposed plans in terms of operating characteristic values and associated risk measures. The results demonstrate reduced inspection effort and testing time while maintaining controlled producer's and consumer's risks. In addition, the methodology is discussed within a broader data-driven and Industry 4.0 setting, highlighting its potential role in supporting modern industrial quality control and supply chain decision processes.

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1. INTRODUCTION

Quality is a fundamental aspect of modern industrial and service systems, significantly influencing customer satisfaction, competitiveness, and long-term organizational success. In manufacturing, a product is considered high quality when it consistently meets functional requirements, performs reliably under expected conditions, and conforms to design standards. Achieving and maintaining such quality levels requires systematic quality control practices throughout production. Two key approaches in quality engineering are statistical process control (SPC) and statistical product control. While SPC focuses on monitoring and controlling process variation, statistical product control ensures that finished products meet specified standards. Acceptance sampling, a major tool within statistical product control, is widely used when full inspection is impractical due to cost, time, or destructive testing. It provides a structured method for deciding whether to accept or reject a production lot based on sample inspection.

An acceptance sampling plan is defined by the sample size n and acceptance number c. A lot is accepted if the

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number of defective items in the sample does not exceed c; otherwise, it is rejected. However, such decisions involve risks: consumer risk (accepting poor-quality lots) and producer risk (rejecting good-quality lots). Designing an efficient sampling plan requires balancing these risks while satisfying quality and contractual requirements. In many applications, product lifetime serves as a critical quality characteristic. Sampling plans based on life testing, known as reliability sampling plans, are used to evaluate whether products meet specified lifetime standards. Typically, the goal is to verify that the average lifetime is at least a predefined value. If the observed data supports this, the lot is accepted; otherwise, it is rejected. For highly reliable products, life testing under normal conditions may take too long. In such cases, truncated life tests are employed, where testing is stopped at a predetermined time or upon reaching a specific number of failures. These tests reduce time and cost while ensuring that the product meets required reliability levels with a specified probability.

2. LIST OF SYMBOLS

n - Sample size
c - Acceptance number
d - Number of defectives
c_1 - Acceptance number based on examining the first quality characteristics.
c_2 - Acceptance number based on examining the second quality characteristics.
d_1, d_2 - Number of defectives based on examining the two quality characteristics
p - Proportion defective
τ - Scale parameter
θ - Shape parameter
μ - Actual mean lifetime
μ_0- Specified mean lifetime

3. FRAMEWORK FOR RELIABILITY ACCEPTANCE SAMPLING PLAN

Reliability acceptance sampling plan (RASP) began with Epstein (1954), who introduced truncated life tests under the exponential model, laying the foundation for integrating lifetime information into sampling decisions. This was followed by Sobel and Tischendorf (1959), who formalized acceptance sampling procedures using new life-test objectives, establishing the earliest structured reliability sampling framework. Goode and Kao (1960) strengthened this direction by proposing Weibull-based sampling plans, while Gupta and Gupta (1961) extended parametric modeling to the Gamma distribution. Gupta (1962) broadened methodological options by introducing Normal and Lognormal lifetime models for life-test sampling. Later, Fertig and Mann (1980) refined Weibull-based reliability sampling by improving estimation under two-parameter Weibull populations. The field then expanded to alternative lifetime distributions: Kantam and Rosaiah (1998) used the Half Logistic distribution, Rosaiah and Kantam (2005) proposed plans for the Inverse Rayleigh distribution, and Kantam et al. (2001) introduced Log Logistic based sampling. Baklizi (2003) developed truncated life-test plans under the Pareto Type-II model, addressing heavy tailed lifetimes, while Rosaiah et al. (2006) proposed plans using the Exponentiated Log-Logistic distribution. Balakrishnan et al. (2007) further enhanced distributional flexibility with sampling plans based on the Generalized Birnbaum Saunders distribution. Rao et al. (2009) developed plans under the Marshall Olkin Extended Exponential model, which they extended in Rao et al. (2011) to incorporate economic considerations. Aslam et al. (2010) contributed time truncated acceptance sampling plans for the Generalized Exponential distribution to support rapid or incomplete life testing. Kannan et al. (2021) introduced sampling plans based on truncated life test under exponentiated half logistic distribution. Kumarasamy and Thottathil (2022) extended this framework to Rayleigh type lifetime models for percentile life estimation; Recent developments focused on Poisson-based approaches: Veerakumari and Mynarani (2024) introduced a bivariate Poisson model with product life time follows Exponentiated Rayleigh distribution to capture dual-stage defect counts and enhance reliability estimation using real industrial data; Kumarasamy et al. (2025) applied the Intervened Poisson Distribution with the Birnbaum Saunders model to represent intervention affected defects.

Under a single sampling plan (SSP) constructed using univariate probability models, a random sample is drawn from the lot and inspected, and the total count of nonconforming units d is noted. The lot is accepted when this count does not exceed the prescribed acceptance threshold c; otherwise, the lot is rejected. When a bivariate model is adopted, the inspection procedure is generalized to simultaneously evaluate two distinct quality characteristics. In this setting, the defect counts d_1 and d_2 associated with the first and second characteristics are recorded, and lot acceptance is granted only when both counts fall within their respective acceptance limits, $d_1 \leq c_1$ and $d_2 \leq c_2$; failure to satisfy either condition results in lot rejection. For such a truncated life test and the associated decision rule, one may be interested in obtaining the smallest sample size to reach a decision.

3.1 Integrating RASP and Industry 4.0 Technologies

The concept of Industry 4.0 has emerged as a transformative paradigm integrating advanced digital technologies into manufacturing and reliability systems. Recent studies, such as Xu et al. (2018), highlight the convergence of digitalization, automation, and data-driven decision-making in modern industrial environments. Building on this, Lee et al. (2015) introduced cyber-physical systems (CPS) architectures that enable real-time interaction between physical

processes and computational models. The role of big data analytics in supporting predictive and prescriptive decision-making has been emphasized by Lu (2017), while Lasi et al. (2014) provided a comprehensive overview of Industry 4.0 components, including smart factories, vertical and horizontal integration, and intelligent automation. The importance of scalable data processing through cloud and edge computing has been discussed by Zhou et al. (2015), and the application of digital twin technology for virtual modeling and system analysis has been explored by Tao et al. (2019). Recent studies have increasingly focused on key Industry 4.0 elements such as predictive analytics, smart inspection systems, and AI-assisted reliability estimation. Predictive analytics enables the anticipation of system failures using historical and real-time data, while smart inspection systems support automated and continuous quality monitoring in manufacturing environments. Furthermore, AI-assisted reliability estimation techniques enhance the accuracy of lifetime modeling and decision-making under uncertainty. In reliability-based acceptance sampling, these elements facilitate dynamic estimation of model parameters, real-time monitoring of failure occurrences within truncated test durations, and automated implementation of decision rules, thereby strengthening the integration of statistical reliability methods within intelligent and data-driven industrial systems.

3.2 Data-Driven Decision Framework for Acceptance Sampling

The suggested methodology is intended to help practitioners make effective lot acceptance decisions while balancing reliability requirements, inspection cost, and risk. It can be understood as a data-driven decision-making model for industrial quality control and reliability management under time-truncated life testing. The implementation of the framework involves the following structured steps:

Step 1: Data Collection and Model Selection
Collect lifetime or failure time data from the production process or field operation. Fit an appropriate lifetime distribution; in this study, the EG distribution is employed due to its flexibility in modeling extreme values and reliability behavior.

Step 2: Specification of Reliability Requirements
Define the desired minimum mean lifetime μ_0, confidence level P^*, and acceptable risk levels from both producer's and consumer's perspectives.

Step 3: Determination of Test Plan Parameters
Select the termination time t for the truncated life test and compute the ratio t/μ_0. Choose the acceptance numbers depending on whether a univariate or bivariate failure model is considered.

Step 4: Sample Size Determination
Using the proposed tables, determine the minimum required sample size n corresponding to the specified design parameters. This step ensures that the reliability requirement is satisfied with the desired confidence level.

Step 5: Decision Rule Implementation
Conduct the life test on n units up to time t. Accept the lot if the observed number of failures does not exceed the acceptance numbers. Otherwise, reject the lot. This provides a clear and implementable decision rule for practitioners.

Step 6: Risk Evaluation and Performance Analysis
Use OC functions to evaluate the probability of lot acceptance under different true mean life values. This allows decision-makers to assess producer's risk and consumer's risk, ensuring a balanced decision process.

A Figure 1 is presented to enhance the comprehension of the operating procedures involved.

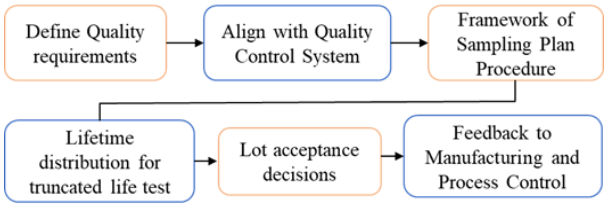


Figure 1. Flowchart of the Reliability Acceptance Sampling Plan Process

3.3 Operating Characteristic function

The probability of acceptance can be regarded as a function of the deviation of the specified average from the true average. This function is called the OC function for the sampling plan. The OC function for the sampling plan $(n, c, t/\mu_0)$ gives the probability of accepting a lot and is given as

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} (1-p)^{n-i} p^i \quad (1)$$

$$L(p) = \sum_{i=0}^{c} \frac{e^{-\lambda} \lambda^i}{i!} \quad (2)$$

The independent Bivariate Poisson distribution is used to simulate two quality criteria concurrently while maintaining statistical simplicity due to the assumption of independence. This formulation extends univariate count models such as binomial and Poisson, allowing the acceptance sampling plan to incorporate numerous defects count in a unifying and practical setting. Campbell (1934) and Aitken (1936) made early contributions to the creation of the Bivariate Poisson Distribution, building on fundamental conceptual insights provided by McKendrick (1925). Irwin (1963) presented a full description of the concept, which was further refined theoretically by Holgate (1964) and Johnson et al. (1969). Furthermore, Hamdan and Al-Bayyati (1969) demonstrated that the Bivariate Poisson Distribution is a limiting instance of the bivariate binomial distribution. The use of bivariate discrete distributions to analyze several quality indicators is widely established Kocherlakota and Kocherlakota (2017). Under the assumption of independent marginals with parameter $\lambda = np$, the joint probability of observing x_1 and x_2 defects across two quality characteristics are

expressed as the product of independent Poisson terms. Consequently, the operating characteristic (OC) function for the proposed bivariate plan is formally given as:

$$L(p) = \sum_{x=0}^{c_1} \frac{e^{-np} np^x}{x!} . \sum_{x=0}^{c_2} \frac{e^{-np} np^x}{x!} \quad (3)$$

For a given t/μ_0 and P^*, the choice of c and n will be made on the basis of the operating characteristics. The sampling plan contains the number of units n on test, an acceptance number c, the maximum test duration t, and the ratio t/μ_0, where μ_0 is the specified average life.

The Exponentiated Gumbel (EG) distribution is chosen for modeling failures, enabling efficient truncated tests and reliable, cost-effective acceptance decisions. Chattamvelli and Shanmugam (2022) studied statistical inference based on the Gumbel distribution. Then the cumulative distribution function and probability density function of Gumbel distribution are

$$F_G(t;\tau) = exp\left[-exp\left(-\frac{t}{\tau}\right)\right], -\infty < t < \infty$$

and

$$f_G(t) = \frac{1}{\tau} e^{t/\tau} exp\left(-e\left(-\frac{t}{\tau}\right)\right), -\infty < t < \infty$$

where τ is a scale parameter. The mean and variance of the Gumbel distribution is

$$E[t] = \tau\gamma$$

and

$$Var[t] = \frac{\pi^2}{6}\tau^2$$

where $\gamma \approx 0.5772156649$ is the Euler – Mascheroni constant. The percentile function of corresponding distribution is

$$Q(p) = \tau ln(-ln(1-p)), \quad 0 < p < 1$$

The survival function and hazard function of Gumbel distribution is denoted as follows

$$S(t) = 1 - F(t) = exp\left(-e^{t/\tau}\right)$$

and

$$h(t) = \frac{f(t)}{S(t)} = \frac{1}{\tau} e^{t/\tau}$$

Acceptance sampling plans for truncated life tests using the Gumbel distribution were studied by Rao et al. (2021). Limited work exists for the EG distribution. This study develops reliability sampling plans under EG distribution, determining minimum sample sizes for specified conditions. The two-parameter Exponentiated Gumbel distribution has the following probability density function.

$$f_{EG}(t;\theta,\tau) = \frac{\theta}{\tau}\left[exp\left(-e^{-t/\tau}\right)\right]^{\theta} e^{-t/\tau}, -\infty < t < \infty \quad (4)$$

Here $\theta > 0$ and $\tau > 0$ are the shape and scale parameters, respectively. The two-parameter EG distribution was originally introduced by Nadarajah (2006) as an extension of the important lifetime model, the Gumbel (1958) distribution, originating from extreme value theory and applied to engineering problems derived by Gumbel. Recently, the EG distribution has received special attention in the probabilistic statistical literature and in various applications. Analytical shapes of the EG distribution, moments, asymptotic distribution of the extreme order statistics, statistical inference and closeness properties relating the EG distribution to some other distributions have been studied by many researchers.

The EG distribution effectively models lifetime data with increasing failure rates, especially for products that wear out rapidly after a certain age. It is more flexible than the standard Gumbel distribution. In series systems, if component lifetimes follow EG, the system lifetime also follows the EG distribution. Nadarajah and Kotz (2006) have established various characteristics of the EG distribution.

Recently, several authors have used the EG distribution in different fields such as reliability, series and parallel systems, climate modeling, rainfall modeling, and material strengths. Kakade et al. (2008) discussed inference for the stress-strength model P ($Y < X$) in the EG distribution. Persson and Ryden (2010) applied the EG distribution for the estimation of return levels of significant wave height. Bagheri et al. (2016) compared several estimators for the EG distribution and showed that the maximum likelihood estimator performs best. Gholami (2016) developed a Bayesian estimation approach for the EG distribution using hierarchical priors and Markov Chain Monte Carlo (MCMC) methods. Khamnei and Mayan (2016) compared parameter estimation for the Gumbel distribution under simple random sampling and ranked set sampling, highlighting the efficiency of ranked set sampling. Dey et al. (2017) examined the properties and frequentist estimation methods of the EG distribution, comparing several estimators through simulations and a flood data application. Fayyazishishavan and Kilic Depren (2021) studied parameter estimation for the two-parameter EG distribution using lower record values via ML and Bayesian methods. Fayomi et al. (2022) proposed a robust generalization of the Gumbel distribution and demonstrated its flexibility and superior performance through theoretical properties, simulations, and real data applications. Chacko and Koshy (2024) studied stress-strength reliability of an s-out-of-k system under the EG distribution using ML and Bayesian methods. Escalante-Sandoval (2025) proposed a bivariate EG model for extreme rainfall and demonstrated its improved performance over existing models. Putri and Sirait (2025) applied the EG distribution to compute premium sufficiency reserves for last-survivor endowment life insurance and found slightly lower reserves compared to the Indonesian Mortality Table 2019. The cumulative distribution function of EG (θ,τ) is given as

$$F_{EG}(t;\theta,\tau) = F_G(t;\tau) = \left[exp\left(-e^{-t/\tau}\right)\right]^{\theta} \quad (5)$$

When the shape parameter θ equals one, the EG distribution as given in (4) reduces to the ordinary Gumbel density with scale parameter τ.

$$u(t) = exp\left(-e^{-t/\tau}\right)$$

$$F(t) = u(t)^{\theta}$$

$$f(t) = F(t)' = \theta u(t)^{\theta-1} u'(t)$$

When differentiate $u(t)$

$$\frac{d}{dt}\left(-e^{-t/\tau}\right) = \frac{1}{\tau} e^{-t/\tau}$$

Let $u(t) = \exp\left(-e^{-t/\tau}\right)$. Then $F(t) = [u(t)]^{\theta}$

$$f(t) = F'(t) = \theta[u(t)]^{\theta-1}u'(t)$$

Since $u'(t) = u(t) \cdot \frac{1}{\tau}e^{-t/\tau}$, multiplying yields

$$f(t) = \theta[u(t)]^{\theta-1} \cdot u(t) \cdot \frac{1}{\tau}e^{-t/\tau} = \frac{\theta}{\tau}[u(t)]^{\theta}e^{-t/\tau}$$

Substituting $u(t)$ back gives Equation (4) directly. The mean and variance of EG distribution is given as

$$E[t] = \tau(\gamma + ln\theta)$$

and

$$Var[t] = \frac{\pi^2}{6}\tau^2$$

To establish the rigorous lifetime behavior of the EG distribution, the reliability (or survival) function $R(t;\theta,\tau)$ is defined as the probability that a system survives beyond time t

$$R(t;\theta,\tau) = 1 - \exp\left(-\theta e^{-t/\tau}\right) \quad (6)$$

The corresponding instantaneous failure rate, or hazard function $h(t;\theta,\tau)$, is obtained by taking the ratio of the probability density function $f_{EG}(t;\theta,\tau)$ to the survival function $R(t;\theta,\tau)$

$$h(t;\theta,\tau) = \frac{\frac{\theta}{\tau}exp(-\theta e^{-t/\tau})e^{-t/\tau}}{1-exp(-\theta e^{-t/\tau})} \quad (7)$$

Since τ is the scale parameter, the shape of the hazard function depends only on θ. For any fixed τ, the EG distribution has an increasing hazard function for all values of θ. Furthermore, the hazard function tends to zero as $t \to 0$ and tends to infinity as $t \to \infty$. The reversed hazard function of the EG distribution is:

$$r(t;\theta,\tau) = \tau^{-1}\left(e^{t/\tau}\right)^{\theta} \quad (8)$$

It is observed that for all values of θ, the reversed hazard function is a decreasing function of t.

3.4 Mean Life Criterion

The mean of the Exponentiated Gumbel distribution is given by

$$\mu = \tau\eta \quad (9)$$

where $\eta = \gamma + ln\theta$ and γ denotes the Euler-Mascheroni constant. Hence, the scale parameter can be expressed as

$$\tau = \frac{\mu}{\eta} \quad (10)$$

Substituting Equation (10) into the cumulative distribution function given in Equation (5), The CDF in terms of the mean life is obtained as

$$F(t) = \left[exp\left(-exp\left(-\frac{\eta t}{\mu}\right)\right)\right]^{\theta} \quad (11)$$

The minimum sample size satisfying the condition (1), (2) and (3) can be determined for any specified values of t/μ_0 and P^*. The probability of failure corresponding to the proposed sampling plan is obtained from Equation (11) as

$$p = \left[exp\left(-exp\left(-\frac{\eta t}{\mu_0(\mu/\mu_0)}\right)\right)\right]^{\theta} \quad (12)$$

The operating characteristics values of the proposed acceptance sampling plan $(n, c, t/\mu_0)$ can be obtained using the procedures described in section 3.3.

3.5 Minimum Sample Size

Consumer risk is defined as the probability of accepting a bad lot, where the true average life $\mu < \mu_0$, and is controlled to be at most $1 - P^*$. Hence, P^* serves as the confidence level, ensuring a high probability of rejecting substandard lots. For a fixed P^*, the sampling plan is described by the triplet $(n, c, t/\mu_0)$. It is assumed that the lot size is very large, validating the use of Binomial, Poisson, or Bivariate Poisson models. Under this setup, lot acceptance corresponds to accepting the hypothesis $\mu \geq \mu_0$. The objective is to determine the smallest integer sample size n.

$$\sum_{i=0}^{c}\binom{n}{i}(1-p_0)^{n-i}p_0^i \geq 1 - P^* \quad (13)$$

$$\sum_{i=0}^{c}\frac{e^{-\lambda}\lambda^i}{i!} \geq 1 - P^* \quad (14)$$

$$\sum_{x=0}^{c_1}\frac{e^{-np}np^x}{x!}.\sum_{x=0}^{c_2}\frac{e^{-np}np^x}{x!} \geq 1 - P^* \quad (15)$$

where $F(t)$, as given in Equation (4), represents the failure probability before time t. Since Equation (2) depends only on the ratio t/μ_0 for each predetermined value of the shape parameter, the experiment needs to specify only this ratio. From Equation (4), it can be shown that the relevant probability is a non-decreasing function of the true average life. Accordingly, the required condition can be obtained, indicating that the true average life exceeds the specified average life and, hence, the lot is accepted as a good lot. The smallest sample sizes satisfying inequalities (13), (14), and (15) are obtained and presented in Tables 1, 2, 3 for t/μ_0 = 2.50, 2.75, 3.00, 3.25, 3.50, 3.75, 4.00, 4.25, and 4.50, with $P^* = 0.75, 0.90, 0.95, 0.99$ for $\theta = 2$.

Table 1. Minimum Sample Size for the specified ratio t/μ_0, confidence level P^*, acceptance number c, θ =2 using binomial approximation.

P^*	c	t/μ_0								
		2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
0.75	1	3	2	2	2	2	2	2	2	2
0.75	2	4	4	4	3	3	3	3	3	3
0.75	3	5	5	5	5	4	4	4	4	4
0.75	4	6	6	6	6	6	5	5	5	5
0.75	5	8	7	7	7	7	6	6	6	6
0.75	6	9	8	8	8	8	8	7	7	7
0.75	7	10	10	9	9	9	9	9	8	8
0.75	8	11	11	10	10	10	10	10	9	9
0.75	9	13	12	12	11	11	11	11	10	10
0.75	10	14	13	13	12	12	12	12	12	11
0.90	1	3	3	3	3	3	2	2	2	2
0.90	2	5	4	4	4	4	4	4	3	3
0.90	3	6	6	5	5	5	5	5	5	4
0.90	4	7	7	7	6	6	6	6	6	6
0.90	5	9	8	8	7	7	7	7	7	7
0.90	6	10	9	9	9	8	8	8	8	8

0.90	7	11	11	10	10	9	9	9	9	9
0.90	8	12	12	11	11	11	10	10	10	10
0.90	9	14	13	12	12	12	11	11	11	11
0.90	10	15	14	14	13	13	13	12	12	12
0.95	1	4	3	3	3	3	3	3	3	2
0.95	2	5	5	4	4	4	4	4	4	4
0.95	3	6	6	6	5	5	5	5	5	5
0.95	4	8	7	7	7	6	6	6	6	6
0.95	5	9	9	8	8	8	7	7	7	7
0.95	6	11	10	9	9	9	8	8	8	8
0.95	7	12	11	11	10	10	10	9	9	9
0.95	8	13	12	12	11	11	11	10	10	10
0.95	9	14	14	13	13	12	12	12	11	11
0.95	10	16	15	14	14	13	13	13	12	12
0.99	1	5	4	4	4	3	3	3	3	3
0.99	2	6	6	5	5	5	5	4	4	4
0.99	3	8	7	7	6	6	6	6	5	5
0.99	4	9	8	8	7	7	7	7	7	6
0.99	5	11	10	9	9	8	8	8	8	7
0.99	6	12	11	11	10	10	9	9	9	9
0.99	7	13	12	12	11	11	10	10	10	10
0.99	8	15	14	13	12	12	12	11	11	11
0.99	9	16	15	14	14	13	13	12	12	12
0.99	10	17	16	16	15	14	14	14	13	13

Table 2. Minimum Sample Size for the specified ratio t/μ_0, confidence level P^*, acceptance number c, θ =2 using Poisson approximation.

P^*	c	t/μ_0								
		2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
0.75	1	4	4	3	3	3	3	3	3	3
0.75	2	5	5	5	5	5	5	5	5	5
0.75	3	8	7	7	7	7	7	7	7	7
0.75	4	9	9	8	8	8	8	8	8	8
0.75	5	10	10	10	10	9	9	9	9	9
0.75	6	12	11	11	11	11	10	10	10	10
0.75	7	13	13	12	12	12	12	12	11	11
0.75	8	14	14	13	13	13	13	13	13	13
0.75	9	16	15	15	14	14	14	14	14	14
0.75	10	17	16	16	16	15	15	15	15	15
0.90	1	5	5	5	5	5	5	5	4	4
0.90	2	7	7	6	6	6	6	6	6	6
0.90	3	9	9	9	9	9	9	8	8	8
0.90	4	11	11	10	10	10	10	10	10	10
0.90	5	12	12	12	12	11	11	11	11	11
0.90	6	14	13	13	13	13	13	12	12	12
0.90	7	15	15	15	14	14	14	14	14	14
0.90	8	17	16	16	16	15	15	15	15	15
0.90	9	18	18	17	17	17	16	16	16	16
0.90	10	20	19	19	18	18	18	17	17	17
0.95	1	6	6	6	6	6	5	5	5	5
0.95	2	8	8	7	7	7	7	7	7	7
0.95	3	11	10	10	10	10	10	10	9	9
0.95	4	12	12	12	11	11	11	11	11	11
0.95	5	14	13	13	13	13	13	12	12	12
0.95	6	15	15	15	14	14	14	14	14	14
0.95	7	17	16	16	16	15	15	15	15	15
0.95	8	19	18	17	17	17	17	16	16	16
0.95	9	20	19	19	18	18	18	18	18	18
0.95	10	21	21	20	20	20	19	19	19	19
0.99	1	8	8	8	8	8	7	7	7	7
0.99	2	10	10	10	10	9	9	9	9	9
0.99	3	13	13	13	12	12	12	12	12	12
0.99	4	15	15	14	14	14	14	14	13	13
0.99	5	17	16	16	16	15	15	15	15	15
0.99	6	19	18	18	17	17	17	17	16	16
0.99	7	20	20	19	19	18	18	18	18	18
0.99	8	22	21	21	20	20	20	20	19	19
0.99	9	24	23	22	22	21	21	21	21	21
0.99	10	25	24	24	23	23	23	22	22	22

Table 3 . Minimum Sample Size for the specified ratio t/μ_0, confidence level P^*, acceptance numbers c_1, c_2, θ =2 using Bivariate Poisson approximation.

P^*	c	t/μ_0								
		2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
0.75	1	2	2	2	2	2	2	2	2	2
0.75	2	4	4	3	3	3	3	3	3	3
0.75	3	5	5	5	4	4	4	4	4	4
0.75	4	6	6	6	6	5	5	5	5	5
0.75	5	7	7	7	7	7	6	6	6	6
0.75	6	8	8	8	8	8	7	7	7	7
0.75	7	10	9	9	9	9	9	8	8	8
0.75	8	11	10	10	10	10	10	9	9	9
0.75	9	12	11	11	11	11	11	11	10	10
0.75	10	13	13	12	12	12	12	12	11	11
0.90	1	3	3	3	3	3	3	3	3	3
0.90	2	5	4	4	4	4	4	4	4	4
0.90	3	6	6	6	6	5	5	5	5	5
0.90	4	7	7	7	7	7	7	6	6	6
0.90	5	9	8	8	8	8	8	8	8	8
0.90	6	10	10	9	9	9	9	9	9	9
0.90	7	11	11	11	10	10	10	10	10	10
0.90	8	12	12	12	11	11	11	11	11	11
0.90	9	14	13	13	13	12	12	12	12	12
0.90	10	15	14	14	14	14	13	13	13	13
0.95	1	4	4	4	4	4	3	3	3	3
0.95	2	5	5	5	5	5	5	5	5	5
0.95	3	7	7	6	6	6	6	6	6	6
0.95	4	8	8	8	8	7	7	7	7	7
0.95	5	10	9	9	9	9	9	8	8	8
0.95	6	11	11	10	10	10	10	10	10	9
0.95	7	12	12	12	11	11	11	11	11	11
0.95	8	14	13	13	12	12	12	12	12	12
0.95	9	15	14	14	14	13	13	13	13	13
0.95	10	16	16	15	15	15	14	14	14	14
0.99	1	5	5	5	5	5	5	5	5	5
0.99	2	4	7	7	7	6	6	6	6	6
0.99	3	8	8	8	8	8	8	7	7	7
0.99	4	10	10	9	9	9	9	9	9	9
0.99	5	11	11	11	11	10	10	10	10	10

0.99	6	13	12	12	12	12	12	11	11	11
0.99	7	14	14	14	13	13	13	13	13	13
0.99	8	16	15	15	15	14	14	14	14	14
0.99	9	17	17	16	16	16	15	15	15	15
0.99	10	19	18	18	17	17	17	16	16	16

3.6 Producers Risk

Producer's risk (α) is defined as the likelihood of rejecting a product lot even though the true mean lifetime μ is greater than the specified mean lifetime μ_0. For a predetermined producer's risk level, the corresponding value of the ratio μ/μ_0 is determined such that the risk associated with the proposed sampling plan does not exceed the specified threshold. Accordingly, the probability p can be expressed as a function of the lifetime ratio μ/μ_0 in the form

$$p = F\left(\frac{t}{\mu_0} \cdot \frac{\mu_0}{\mu}\right)$$

The required value of μ/μ_0 is obtained as the smallest positive solution satisfying

$$P_a(p) = 1 - \alpha \qquad (16)$$

For a given sampling plan characterized by $(n, c, t/\mu_0)$ and a specified confidence level P^*, the minimum values of μ/μ_0 that fulfill this condition are examined. This ensures that acceptable lots are not unnecessarily rejected, while maintaining effective control over producer's risk.

4. CONSTRUCTION OF THE TABLE

- The values $t/\mu_0 = 2.5, 2.75, 3, 3.25, 3.5, 3.75, 4, 4.25$ and 4.5 together with $\theta = 2$ and 5, are substituted into Equation (5).
- Using these substitutions, the minimum sample size n is identified such that the requirements imposed by Equations (13), (14), and (15) are simultaneously satisfied.
- For the obtained sample size n, the corresponding minimum test ratio is computed by applying Equation (12).
- The OC are then evaluated by inserting the relevant parameter values into Equations (1), (2), and (3).

5.COMPUTATIONAL VALIDATION OF THE PROPOSED METHODOLOGY

A Monte Carlo simulation-based computational study was conducted to evaluate the adequacy and robustness of the Exponentiated Gumbel distribution for modeling machine failure time data (in hours). The observed/simulated failure times are: 1736, 2565, 1903, 2889, 3242, 1283, 2071, 2933, 2106, 1968, 3407, 1964, 2318, 2139, 1436, 2971, 1678, 1270, 1792, 3380, 2919, 2348, 2251, 4426, 2278, 2379, 2095, 2173, 1739, 1521, 3486, 2984, 2344, 2584, 1192, 1998, 2489, 1634, 1779, 1656, which exhibit a typical characteristic of industrial lifetime data. The goodness-of-fit analysis based on maximum likelihood estimation yielded a log-likelihood of −315.7602, with AIC = 637.5204 and BIC = 642.587, indicating a satisfactory balance between model fit and complexity. Further, the Kolmogorov–Smirnov test produced a statistic of D = 0.07814 with a p-value of 0.9516, suggesting no significant difference between the empirical and theoretical distributions, thereby confirming the adequacy of the proposed model. In addition, a sensitivity analysis was performed through Monte Carlo simulations by varying the parameters θ and τ, where results showed that increasing τ from 300 to 700 substantially increases the variance, while variations in θ influence the skewness and early failure behaviour of the distribution. These findings confirm that τ controls the dispersion of failure times, whereas θ governs the shape characteristics of the distribution. Overall, the combined evidence from Monte Carlo simulation, goodness-of-fit measures, and sensitivity analysis supports that the proposed EG distribution is statistically adequate, computationally stable, and flexible for modeling machine failure time data.

Consider a reliability experiment in which the lifetime distribution follows an EG distribution with parameter $\theta = 2$. The objective is to test the null hypothesis that the true mean lifetime μ meets or exceeds a specified value μ_0 =1000 hours, subject to a confidence level $P^* = 0.90$, under a time-truncated life test terminated at $t = 2500$ hours ($t/\mu_0 = 2.5$). For an acceptance number $c = 2$, the minimum sample size corresponding to $P^* = 0.90$, $t/\mu_0 = 2.5$, and $c = 2$ is obtained from Table 1 as $n = 5$. The resulting sampling plan is therefore given by $(n, c, t/\mu_0) = (5,2,2.5)$. The associated decision function is defined as: accept the lot if the number of observed failures within time t does not exceed c; otherwise, reject the lot. The operating characteristic (OC) values for the binomial-based sampling plan are obtained from Table 4, and the corresponding OC curve is depicted in Figure 2. For comparison, the Poisson approximation yields the sampling plan $(n = 7, c = 2, t/\mu_0 = 2.5)$, with OC values reported in Table 5. It is observed that for $\mu/\mu_0 = 2$, the producer's risk is approximately 0.6183, indicating that higher quality levels are required to achieve higher acceptance probabilities under this design.

Under the Bivariate Poisson framework with acceptance numbers $c_1 = 2$and $c_2 = 2$, and confidence level $P^* = 0.75$, the minimum ratio μ/μ_0 required to bound the producer's risk at 0.05 is obtained from Table 3 as 93.41. The corresponding sampling plan is $(n, c_1, c_2, t/\mu_0) = (5,2,2,2.5)$, with the associated operating characteristic values summarized in Table 6. Comparative results presented in Table 7 indicate that the acceptance probability is monotonically increasing in μ/μ_0, while larger sample sizes yield steeper OC curves, thereby enhancing discriminatory power between acceptable and non-acceptable quality levels. Furthermore, for a fixed sample size, the Binomial model yields higher

acceptance probabilities than the Bivariate Poisson model, reflecting the additional variability induced by dependence between failure modes. This behavior is further illustrated in.

Table 4. Values of the OC function of the sampling plan $(n, c, t/\mu_0)$ for given confidence level P^* with $\theta = 2$ using binomial approximation.

P^*	n	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	4	2.50	0.5863	0.8803	0.9388	0.9589	0.9686	0.9739
0.75	4	2.75	0.5195	0.8579	0.9290	0.9536	0.9649	0.9713
0.75	4	3.00	0.4548	0.8335	0.9184	0.9476	0.9611	0.9686
0.75	3	3.25	0.6932	0.9303	0.9696	0.9817	0.9869	0.9898
0.75	3	3.50	0.6475	0.9181	0.9649	0.9793	0.9855	0.9887
0.75	3	3.75	0.6016	0.9046	0.9598	0.9766	0.9839	0.9876
0.75	3	4.00	0.5604	0.8901	0.9541	0.9738	0.9821	0.9865
0.75	3	4.25	0.5116	0.8743	0.9479	0.9707	0.9803	0.9852
0.75	3	4.50	0.4687	0.8575	0.9413	0.9673	0.9782	0.9839
0.90	5	2.50	0.3817	0.7759	0.8772	0.9151	0.9336	0.9443
0.90	4	2.75	0.5195	0.8579	0.9291	0.9536	0.9649	0.9713
0.90	4	3.00	0.4548	0.8335	0.9184	0.9476	0.9611	0.9686
0.90	4	3.25	0.3936	0.8071	0.9067	0.9411	0.9569	0.9656
0.90	4	3.50	0.3369	0.7789	0.8939	0.9341	0.9524	0.9624
0.90	4	3.75	0.2858	0.7492	0.8803	0.9265	0.9476	0.9589
0.90	4	4.00	0.2403	0.7182	0.8659	0.9184	0.9424	0.9555
0.90	3	4.25	0.5116	0.8743	0.9479	0.9707	0.9803	0.9852
0.90	3	4.50	0.4687	0.8575	0.9413	0.9673	0.9782	0.9839
0.95	5	2.50	0.3817	0.7759	0.8772	0.9151	0.9336	0.9443
0.95	5	2.75	0.3122	0.7398	0.8549	0.9046	0.9266	0.9391
0.95	4	3.00	0.4548	0.8335	0.9184	0.9476	0.9611	0.9689
0.95	4	3.25	0.3936	0.8071	0.9067	0.9411	0.9569	0.9656
0.95	4	3.50	0.3369	0.7789	0.8939	0.9341	0.9524	0.9624
0.95	4	3.75	0.2858	0.7492	0.8803	0.9265	0.9476	0.9589
0.95	4	4.00	0.2403	0.7182	0.8657	0.9184	0.9424	0.9555
0.95	4	4.25	0.2003	0.6861	0.8501	0.9097	0.9369	0.9516
0.95	4	4.50	0.1658	0.6532	0.8335	0.9005	0.9311	0.9476
0.99	6	2.50	0.2329	0.6615	0.8019	0.8588	0.8877	0.9047
0.99	6	2.75	0.1749	0.6149	0.7764	0.8428	0.8766	0.8964
0.99	5	3.00	0.2509	0.7018	0.8405	0.8934	0.9189	0.9336
0.99	5	3.25	0.1985	0.6622	0.8202	0.8814	0.9109	0.9278
0.99	5	3.50	0.1547	0.6216	0.7986	0.8685	0.9025	0.9216
0.99	5	3.75	0.1899	0.5804	0.7759	0.8549	0.8934	0.9151
0.99	4	4.00	0.2403	0.7182	0.8657	0.9184	0.9424	0.9551
0.99	4	4.25	0.2003	0.6861	0.8501	0.9097	0.9369	0.9516
0.99	4	4.50	0.1658	0.6532	0.8334	0.9005	0.9311	0.9476

Table 5. Values of the operating characteristic function of the sampling plan $(n, c = 2, t/\mu_0)$ for given confidence level P^* with $\theta = 2$ using Poisson approximation.

P^*	n	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	5	2.50	0.4649	0.7535	0.8483	0.8886	0.9097	0.9223
0.75	5	2.75	0.4197	0.7228	0.8307	0.8772	0.9015	0.9161
0.75	5	3.00	0.3799	0.6919	0.8122	0.8652	0.8930	0.9097
0.75	5	3.25	0.3451	0.6609	0.7932	0.8526	0.8841	0.9029
0.75	5	3.50	0.3150	0.6302	0.7735	0.8396	0.8748	0.8959
0.75	5	3.75	0.2889	0.6001	0.7535	0.8261	0.8652	0.8886
0.75	5	4.00	0.2666	0.5708	0.7331	0.8122	0.8552	0.8810
0.75	5	4.25	0.2474	0.5426	0.7125	0.7980	0.8449	0.8732
0.75	5	4.50	0.2309	0.5154	0.6919	0.7834	0.8343	0.8652
0.90	7	2.50	0.2460	0.5698	0.7111	0.7781	0.8152	0.8383
0.90	7	2.75	0.2074	0.5284	0.6831	0.7585	0.8007	0.8270
0.90	6	3.00	0.2625	0.5874	0.7350	0.8044	0.8422	0.8654
0.90	6	3.25	0.2313	0.5515	0.7107	0.7877	0.8300	0.8559
0.90	6	3.50	0.2052	0.5168	0.6860	0.7705	0.8174	0.8462
0.90	6	3.75	0.1833	0.4835	0.6613	0.7529	0.8044	0.8362
0.90	6	4.00	0.1651	0.4518	0.6365	0.7350	0.7911	0.8258
0.90	6	4.25	0.1498	0.4219	0.6118	0.7168	0.7774	0.8152
0.90	6	4.50	0.1370	0.3938	0.5874	0.6984	0.7635	0.8044
0.95	8	2.50	0.1724	0.4832	0.6388	0.7167	0.7611	0.7893
0.95	8	2.75	0.1402	0.4399	0.6070	0.6937	0.7437	0.7754
0.95	7	3.00	0.1758	0.4884	0.6548	0.7385	0.7857	0.8152
0.95	7	3.25	0.1502	0.4502	0.6263	0.7180	0.7703	0.8032
0.95	7	3.50	0.1294	0.4141	0.5979	0.6971	0.7546	0.7908
0.95	7	3.75	0.1125	0.3802	0.5698	0.6760	0.7385	0.7781
0.95	7	4.00	0.0988	0.3487	0.5420	0.6548	0.7221	0.7651
0.95	7	4.25	0.0876	0.3196	0.5149	0.6334	0.7055	0.7519
0.95	7	4.50	0.0784	0.2928	0.4884	0.6121	0.6887	0.7385
0.99	10	2.50	0.0802	0.3343	0.4997	0.5921	0.6479	0.6844
0.99	10	2.75	0.0606	0.2926	0.4639	0.5641	0.6257	0.6663
0.99	10	3.00	0.0463	0.2550	0.4293	0.5363	0.6033	0.6479
0.99	10	3.25	0.0359	0.2215	0.3960	0.5088	0.5809	0.6294
0.99	9	3.50	0.0478	0.2516	0.4349	0.5504	0.6231	0.6712
0.99	9	3.75	0.0393	0.2221	0.4043	0.5248	0.6023	0.6541
0.99	9	4.00	0.0328	0.1958	0.3751	0.4996	0.5815	0.6369
0.99	9	4.25	0.0277	0.1726	0.3473	0.4749	0.5608	0.6196
0.99	9	4.50	0.0238	0.1521	0.3210	0.4507	0.5402	0.6023

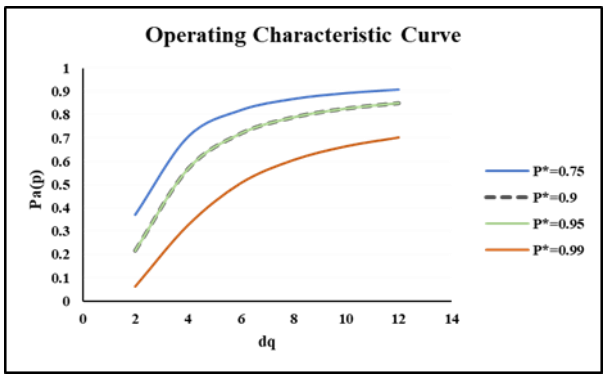


Figure 2. OC curves for the sampling plan $(n, c_1 = 2, c_2 = 2, t/\mu_0 = 2.50)$ based on mean life of EG distribution under Bivariate poisson approximation with various P^*.

Table 6. Values of the OC function of the sampling plan $(n, c_1, c_2 = 2, t/\mu_0)$ for given confidence level P^* with θ =2 using Bivariate Poisson approximation.

P^*	n	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	4	2.50	0.3696	0.7062	0.8215	0.8701	0.8954	0.9105
0.75	4	2.75	0.3209	0.6692	0.8000	0.8563	0.8857	0.9031
0.75	3	3.00	0.4876	0.7862	0.8807	0.9184	0.9371	0.9479
0.75	3	3.25	0.4494	0.7600	0.8666	0.9097	0.9312	0.9436
0.75	3	3.50	0.4153	0.7333	0.8517	0.9005	0.9249	0.9390
0.75	3	3.75	0.3852	0.7065	0.8361	0.8909	0.9184	0.9342
0.75	3	4.00	0.3589	0.6796	0.8200	0.8807	0.9115	0.9291
0.75	3	4.25	0.3358	0.6530	0.8033	0.8702	0.9042	0.9239
0.75	3	4.50	0.3158	0.6269	0.7862	0.8592	0.8967	0.9184
0.90	5	2.50	0.2161	0.5677	0.7197	0.7896	0.8275	0.8507
0.90	4	2.75	0.3209	0.6692	0.8000	0.8563	0.8857	0.9031
0.90	4	3.00	0.2793	0.6318	0.7776	0.8418	0.8754	0.8954
0.90	4	3.25	0.2442	0.5948	0.7544	0.8267	0.8647	0.8873
0.90	4	3.50	0.2147	0.5584	0.7306	0.8109	0.8535	0.8789
0.90	4	3.75	0.1901	0.5230	0.7062	0.7945	0.8418	0.8701
0.90	4	4.00	0.1695	0.4889	0.6816	0.7776	0.8298	0.8610
0.90	4	4.25	0.1524	0.4565	0.6567	0.7603	0.8173	0.8516
0.90	4	4.50	0.1381	0.4257	0.6318	0.7426	0.8044	0.8418
0.95	5	2.50	0.2161	0.5677	0.7197	0.7896	0.8275	0.8507
0.95	5	2.75	0.1762	0.5225	0.6900	0.7694	0.8127	0.8393
0.95	5	3.00	0.1443	0.4787	0.6597	0.7485	0.7974	0.8275
0.95	5	3.25	0.1191	0.4367	0.6291	0.7270	0.7816	0.8152
0.95	5	3.50	0.0992	0.3971	0.5984	0.7049	0.7653	0.8026
0.95	5	3.75	0.0835	0.3601	0.5677	0.6825	0.7485	0.7896
0.95	5	4.00	0.0711	0.3258	0.5375	0.6597	0.7313	0.7762
0.95	5	4.25	0.0612	0.2944	0.5077	0.6368	0.7138	0.7625
0.95	5	4.50	0.0533	0.2656	0.4787	0.6137	0.6960	0.7485
0.99	7	2.50	0.0605	0.3246	0.5056	0.6054	0.6646	0.7028
0.99	7	2.75	0.0430	0.2792	0.4666	0.5754	0.6411	0.6839
0.99	7	3.00	0.0309	0.2385	0.4287	0.5453	0.6174	0.6646
0.99	6	3.25	0.0535	0.3042	0.5051	0.6204	0.6889	0.7326
0.99	6	3.50	0.0421	0.2671	0.4707	0.5937	0.6681	0.7161
0.99	6	3.75	0.0336	0.2338	0.4373	0.5669	0.6471	0.6992
0.99	6	4.00	0.0273	0.2041	0.4051	0.5402	0.6258	0.6820
0.99	6	4.25	0.0224	0.1780	0.3743	0.5138	0.6044	0.6646
0.99	6	4.50	0.0188	0.1550	0.3450	0.4877	0.5829	0.6471

Table 7 presents the OC values of the proposed Binomial, Poisson, and Bivariate Poisson sampling plans for different values of θ. The probability of acceptance increases monotonically with μ/μ_0, indicating higher acceptance for better-quality lots. Larger sample sizes produce steeper OC curves, offering improved discrimination between poor- and good-quality lots. Figure 3 illustrates this behavior, confirming the robustness of the proposed plans under practical industrial parameter settings and Figure 4 presents OC curve for the sampling plan.

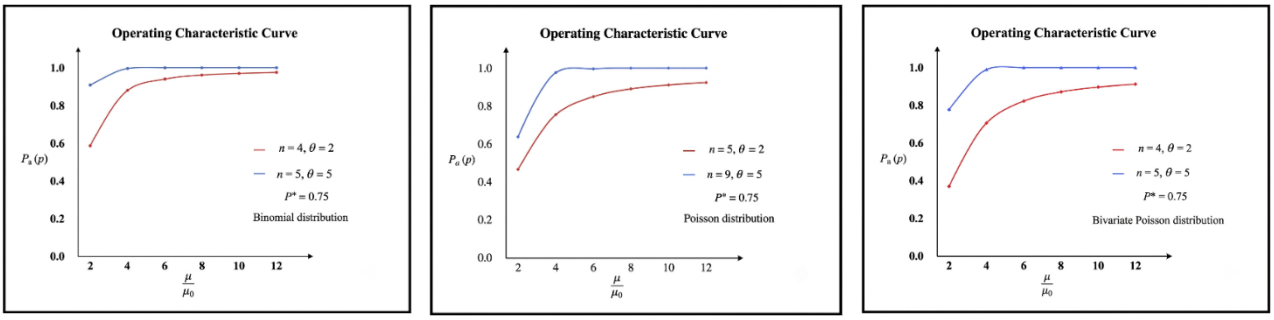


Figure 3. OC curves for the sampling plan $(n, c = 2, c_1 = 2, c_2 = 2, t/\mu_0 = 2.5)$ based on mean life of EG distribution under binomial, Poisson and bivariate Poisson approximation with θ = 2 and 5.

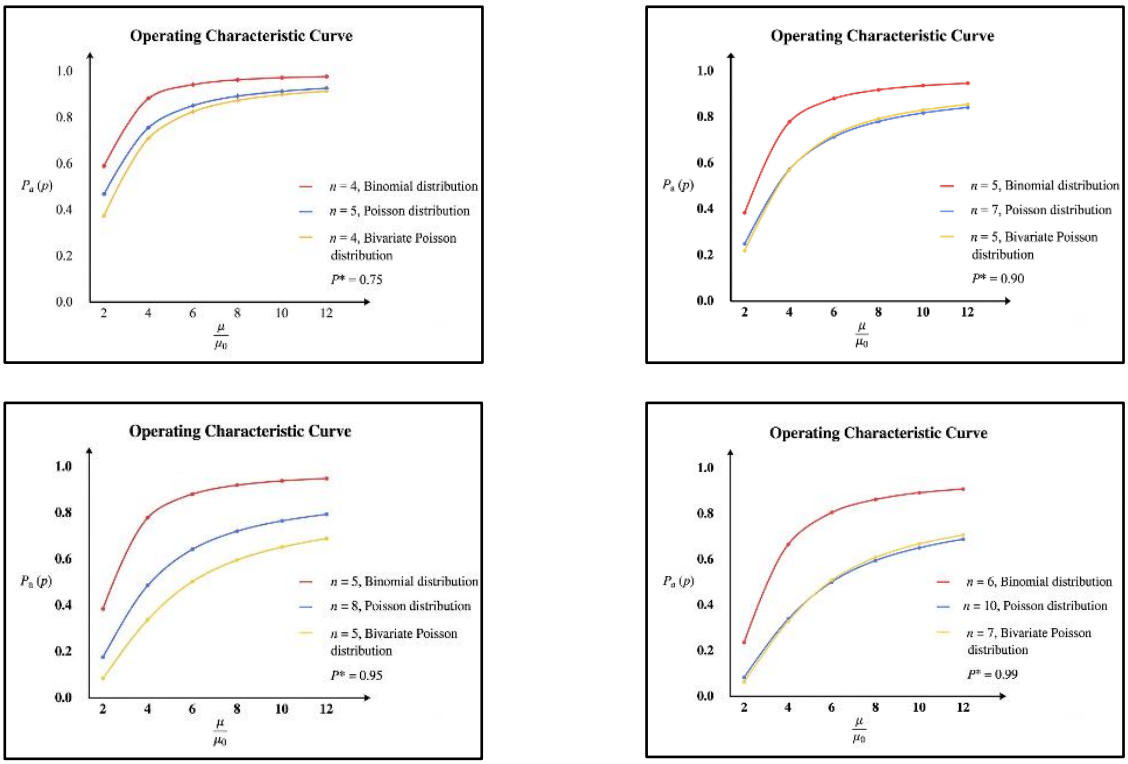


Figure 4. OC curve for the sampling plan $(n, c_1 = 2, c_2 = 2, t/\mu_0 = 2.5)$ based on mean life of EG distribution under binomial, Poisson and bivariate Poisson approximation with varying P^*.

Table 7. OC values for the sampling plan $(n, c = c_1 = c_2 = 2, t/\mu_0 = 2.5)$ based on mean life of EG distribution under binomial, Poisson and bivariate Poisson approximation with $\theta = 2$ and 5.

Distribution	θ	n	μ/μ_0					
			2	4	6	8	10	12
Binomial distribution	2	4	0.5862	0.8803	0.9388	0.9590	0.9685	0.9739
	5	5	0.9080	0.9971	0.9995	0.9998	0.9999	0.9999
Poisson distribution	2	5	0.4649	0.7535	0.8483	0.8886	0.9097	0.9223
	5	9	0.6366	0.9749	0.9952	0.9982	0.9991	0.9994
Bivariate Poisson distribution	2	4	0.3696	0.7062	0.8215	0.8701	0.8954	0.9105
	5	5	0.7760	0.9895	0.9982	0.9994	0.9997	0.9998

Table 8. Goodness-of-fit measures for Example 1 dataset.

	Kolmogorov – Smirnov Test				
Distribution	Test statistics	p - value	Log - likelihood	AIC	BIC
Exponentiated Gumbel	0.2141	0.6121	-65.443	134.886	135.682
Exponentiated Rayleigh	0.2071	0.6610	-64.536	133.073	133.868

Table 9. Goodness-of-fit measures for Example 2 dataset.

	Kolmogorov – Smirnov Test				
Distribution	Test statistics	p - value	Log -likelihood	AIC	BIC
Exponentiated Gumbel	0.0634	0.8651	-143.765	291.530	296.582
Exponentiated Rayleigh	0.0662	0.8351	-138.141	280.282	285.237

For the same sample size n, OC curves show the Binomial model yields higher acceptance probabilities than the Bivariate Poisson model across all μ/μ_0 and confidence levels. The Bivariate Poisson accounts for dependence between characteristics, adding variability and making the plan more stringent, thus providing a more conservative and realistic quality assessment.

Case Study 1: Aircraft Air-Conditioning System

To illustrate the practical applicability of the proposed method, consider failure time data of an aircraft air-conditioning system reported by Bain and Engelhardt (1991). 55, 56, 104, 176, 182, 220, 239, 246, 320. Based on Table 8 and Figure 5, the Exponentiated Gumbel distribution provides an adequate fit to the data. The Kolmogorov-Smirnov (K-S) test statistic is 0.2141 with a corresponding p-value of 0.6121, indicating no statistical evidence to reject the null hypothesis that the data follow an Exponentiated Gumbel distribution. Although the Exponentiated Rayleigh distribution shows a slightly lower AIC (133.073 vs. 134.886) and BIC (133.868 vs. 135.682) and a slightly smaller KS statistic, the Exponentiated Gumbel adequately captures the data, including extreme values, and is therefore considered the most suitable distribution for this dataset. When two independent quality characteristics contribute to failures in an airplane's air conditioning system during repeated operation, a Bivariate Poisson model is suitable.

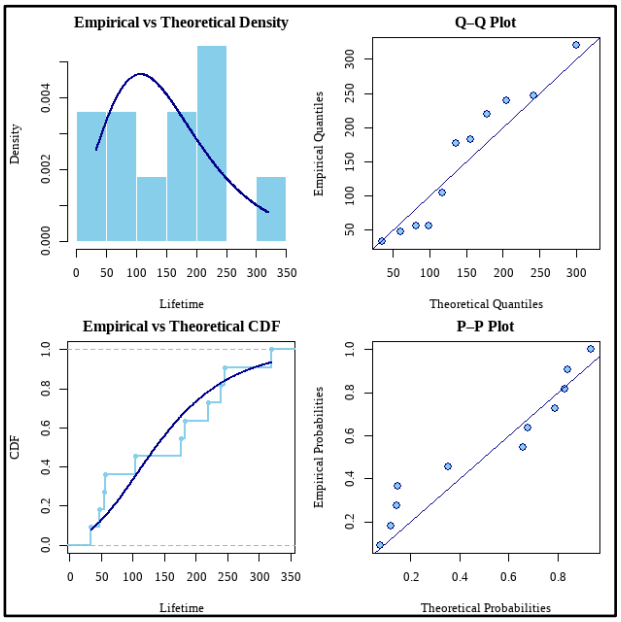


Figure 5. Graphical representation of an Aircraft Air-Conditioning System.

Failures are categorized into two components: Q_1, representing failures caused by compressor malfunctions during operational cycles, and Q_2, representing failures due to coolant leaks. Here we have to establish a sampling plan to test whether the average failure time of the air conditioning system is at least μ_0 = 30 hours with a predetermined confidence level P^* = 0.75 and we wish to stop the test at t = 75 hours or equivalently t/μ_0 = 2.5. Thus, for an acceptance number c_1 = 2 based on the compressor malfunction and c_2 = 2 based on the coolant leaks, confidence level P^*= 0.75, the required n is found from Table 3 based on the proposed plan. In this case, the reliability sampling plan for truncated life tests from EG distribution is $(n, c_1, c_2, t/\mu_0)$ = (4,2,2,2.5). The lot is accepted if the number of failures due to each cause does not exceed 2 within 45 hours; since only one failure is observed during the test period, the lot is accepted.

Case Study 2: Windshield Failure time Data

Consider a problem associated with validating the average failure time of the windshields from Murthy et al. (2004). 0.04, 0.3, 0.31, 0.557, 0.943, 1.07, 1.124, 1.248, 1.281, 1.281, 1.303, 1.432, 1.48, 1.51, 1.51, 1.568, 1.615, 1.619, 1.652, 1.652, 1.757, 1.795, 1.866, 1.876, 1.899, 1.911, 1.912, 1.9141, 0.981, 2.010, 2.038, 2.085, 2.089, 2.097, 2.135, 2.154, 2.190, 2.194, 2.223, 2.224, 2.23, 2.3, 2.324, 2.349, 2.385, 2.481, 2.610, 2.625, 2.632, 2.646, 2.661, 2.688, 2.823, 2.89, 2.9, 2.934, 2.962, 2.964, 3, 3.1, 3.114, 3.117, 3.166, 3.344, 3.376, 3.385, 3.443, 3.467, 3.478, 3.578, 3.595, 3.699, 3.779, 3.924, 4.035, 4.121, 4.167, 4.240, 4.255, 4.278, 4.305, 4.376, 4.449, 4.485, 4.570, 4.602, 4.663, 4.694.

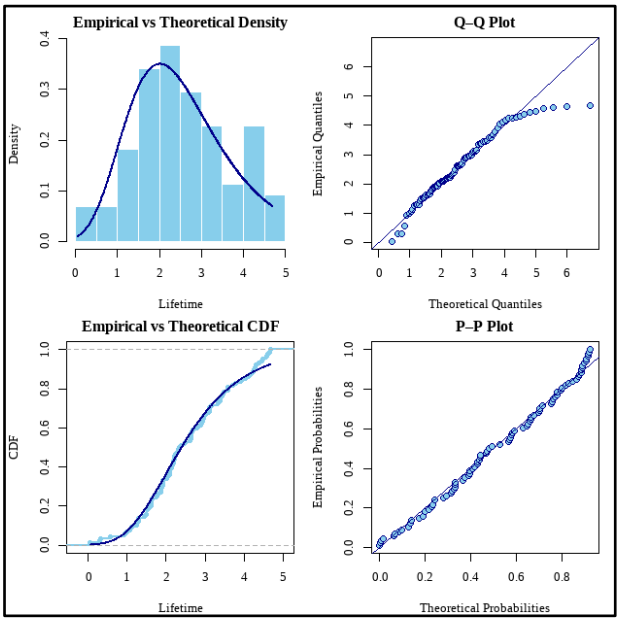


Figure 6. Graphical representation for Windshield Failure Times

Based on Table 9, both the Exponentiated Gumbel and Exponentiated Rayleigh distributions provide a good fit to Dataset 2. The Exponentiated Gumbel has a K-S statistic of 0.0634 with a p-value of 0.8651, indicating strong agreement with the observed data. Although the Exponentiated Rayleigh distribution shows slightly lower AIC (280.282 vs. 291.530) and BIC (285.237 vs. 296.582) values, the Exponentiated Gumbel demonstrates slightly better goodness-of-fit according to the KS test and effectively captures the behavior of the

data, including extreme values. Therefore, the Exponentiated Gumbel distribution is considered the most appropriate model for this dataset which is also shown in Figure 6. We need to establish a sampling plan to test whether the system's average failure time is at least $\mu_0 = 1.259$ units with a predetermined confidence level $P^* = 0.75$. For this purpose, we adopt a univariate approach, considering both Poisson and Binomial distributions to model the occurrence of failures and guide the acceptance sampling procedure. We plan to stop the test at $t = 3.148$ units which corresponds to the desired extended truncation ratio of $t/\mu_0 = 2.50$. Assuming the design criteria for the Exponentiated Gumbel (EG) distribution tables hold for the confidence level $P^* = 0.75$ and acceptance number $c = 5$, the required sample size n is found to be 8. In this scenario, the reliability sampling plan for truncated life tests from the EG distribution is $(n, c, t/\mu_0)$ = (8, 5, 2.50). In the full sample of 90 observations, we have recorded 62 failure times before the stop time of t = 3.148 units. Therefore, since the observed number of failures 62 is greater than the acceptance number $c = 5$, we must reject the lot. We cannot accept the assertion that the average failure time is at least 1.259 units with a confidence level of $P^* = 0.75$.

6. QUANTIFICATION OF OPERATIONAL AND ECONOMIC IMPACTS AND SUPPLY CHAIN DECISION INTEGRATION OF THE PROPOSED MODEL

The proposed RASPs under the Exponentiated Gumbel distribution show clear improvements in both operational efficiency and economic performance. In the developed framework, the sample size is determined as the minimum value n satisfying the specified producer's and consumer's risk constraints based on the OC function. This ensures adequate protection for both producer and consumer while reducing inspection effort. By replacing complete inspection with optimized sampling based on n, the proposed method reduces testing effort and resource usage. The incorporation of truncated life testing further improves efficiency by limiting the test duration to a specified multiple of the mean life, thereby reducing overall inspection time compared to full life testing. From a risk perspective, the proposed plan maintains the consumer's risk within acceptable limits while balancing producer's risk. This reduces the probability of accepting poor-quality lots and rejecting acceptable ones, thereby lowering downstream failures and related costs.

Overall, the proposed approach achieves a balanced trade-off between cost and reliability, making it suitable for industrial applications. The proposed RASP model is not limited to quality control but serves as a decision-support tool within operations and supply chain management systems. It provides a quantitative basis for integrating reliability information into key managerial decisions such as production planning, procurement, and supplier evaluation. By determining optimal sample sizes and test durations, the model supports efficient allocation of inspection resources while minimizing production delays.

Within procurement and supplier selection, the model facilitates objective accept-reject decisions for incoming lots based on reliability performance, ensuring that only components meeting predefined quality standards enter the production system. At the supply chain level, it contributes to quality assurance by preventing the propagation of defective items to downstream stages, thereby reducing disruptions, returns, and warranty-related costs.

Moreover, by incorporating producer's and consumer's risk, the framework enables a structured trade-off between cost and quality objectives, aligning operational decisions with service-level requirements. In this way, the proposed model functions as an integrated decision-making mechanism that enhances coordination and improves overall supply chain efficiency.

7. CONCLUSION

This study developed reliability acceptance sampling plans for truncated life tests under Binomial, Poisson, and Bivariate Poisson models, assuming that product lifetimes follow an EG distribution. The proposed methodology provides systematically constructed tables of minimum sample sizes and corresponding acceptance criteria for various confidence levels, acceptance numbers, and test duration ratios. These plans enable practitioners to make efficient and informed lot acceptance or rejection decisions, particularly in situations where complete life testing is infeasible. The integration of both univariate and bivariate count models enhances modeling flexibility and allows for a more realistic representation of dependence structures in defect occurrences. The use of Monte Carlo simulation further validates the performance of the proposed plans by evaluating operating characteristic values and associated risk measures. The results demonstrate that the proposed approach achieves a desirable balance between producer's and consumer's risks while significantly reducing inspection effort, testing time, and associated operational costs. From a practical perspective, the developed sampling schemes contribute to improved inspection efficiency, faster decision-making, and reduced risk of accepting substandard products or rejecting good-quality lots. Furthermore, the proposed design presents the possibility of integration with automated quality monitoring systems and real-time decision support in supply chain operations within the scope of data-driven manufacturing and Industry 4.0 conditions. In order to account for intervention effects or structural changes in defect patterns, the framework can be expanded for future studies to include the Bivariate Intervened Poisson distribution. Furthermore, investigating different lifetime models like the Exponentiated Rayleigh distribution would expand the

methodology's application to a wider range of industrial reliability challenges.

Data Availability Statement:
The authors confirm that the data supporting the findings of the study are available within the articles 'Statistical Analysis of Reliability and Life Testing Models, Theory and Methods' by Bain and Engelhardt (1991) DOI: 10.2307/2982899 and 'Weibull models' by (Murthy et al. 2004) DOI: 10.2307/27590641

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